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Assessing the Estimands and Estimates of Hospitalization Rates in Health Economics and Clinical Medicine
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ABSTRACT

Even though data on hospital admissions are widely used in health research, hospitalization-related quantities measured using these data are not always clearly conceptualized. Consequently, estimators of these quantities can have unclear rationales and undesirable properties. We evaluate three rate estimators for measuring hospitalization-related quantities that are of interest in health economics and clinical medicine subspecialities. Using the Grossman human capital model, we motivate the importance of measuring healthy time. We show that an upper bound on healthy time can be calculated using lengths of hospital stay without assumptions about health status outside the hospital. We find that an admission rate conventionally used in clinical research is a patient follow-up time weighted average that lacks a clear basis for the weights. We evaluate the Centers for Medicare and Medicaid Services (CMS) use of risk-standardized readmission rates to penalize hospitals under the Hospital Readmissions Reduction Program (HRRP) and find that it may inadvertently conflict with disease-specific care aimed at reducing mortality risk. We show that risk-standardized rates can be sensitive to patient case mix, potentially leading to hospital rankings that do not reflect hospital quality. We also summarize debates regarding the effectiveness of risk-standardized readmission rates in reducing readmissions.

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1. Introduction

Health, administrative, and financial data on hospital admissions and spells offer a rich source of information for answering questions that could improve clinical practice and public health policy. Indeed, researchers in applied health measure various hospitalization-related rates. Hospital admission and readmission rates are used in clinical research to understand disease progression. Readmission rates are tied to national quality metrics and reimbursement. Bed occupancy rates are used to assess hospital performance. Health economists use hospitalization data to understand the economic risk of adverse health shocks and measure utilization-based time allocation outcomes. The costs of providing care, including those for inpatient admissions, are used to measure the burden of disease on healthcare system resources. Costs borne by patients estimate the financial burden on individuals.

In practice, algebraic measures related to hospitalization have been computed without clearly conceptualizing the estimand of interest. Consider the hospital admission rate conventionally measured in clinical research. Written formally, this is

$$\frac{365 \sum_i n_i}{\sum_i d_i},$$

where the summations are over all patients i in a patient cohort observed in a specified time period, n_i denotes the number of hospitalizations for patient i and d_i is the number of days that patient i is observed. Section 3 will show that this rate is equivalent to a weighted average of individual hospitalization rates, with weights proportional to the time a patient is observed in the data. Metcalfe et al. (2003) note that studies heuristically use this definition “to counter" variable follow-up lengths in the patient cohort. We will argue that the substantive rationale for using this weighting scheme when defining the admission rate is still unclear.

The lack of satisfactory appraisals for hospitalization rates reported in medical and public health research motivates us to describe and compare useful rates for measuring important hospitalization-related quantities in health economics and clinical medicine subspecialities. The same name sometimes refers to rates for measuring different quantities. For instance, “readmission rates” used in clinical research on disease progression and hospital performance assessment are defined differently. This makes it especially important to discuss the reasoning and motivation behind defining an estimator for a quantity of interest.

In Section 2, we discuss healthy time as an estimand of interest in health economic study of health capital and production. We propose a hospitalization rate that estimates an upper bound on healthy time. Section 3 discusses a common definition of admission rates used to study disease progression in clinical research. We propose an alternative admission rate that appropriately counts patients who are observed for varying amounts of time. Section 4 evaluates the Centers for Medicare and Medicaid Services (CMS) use of risk-standardized 30-day readmission rates in the Hospital Readmissions Reduction Program (HRRP). We contrast the program’s broad aim with the disease-specific clinician perspective, highlight potential issues with using standardized readmission ratios to calculate hospital payment reduction, summarize debates about the methodology used by

CMS, and discuss the program's possible impact on provider behavior and readmission trends.

To our knowledge, no existing research in clinical medicine or health economics has reviewed or interpreted the definitions of alternative hospitalization rates for hospital utilization data. In clinical medicine, the need for "adjustment" of individual admission rates when the follow-up times differ between groups of patients is a recognized problem (Metcalfe et al., 2003). Our proposed admission rate aggregates patient-specific rates rather than using a cohort-wide rate. A second new contribution is our use of admissions data to estimate an upper bound on the distribution of individual healthy time. This responds to the need cited by Burns and Mullahy (2016) for "downstream research" on measurement of health status when individuals are not in contact with the healthcare system.

2. Health Economics: Health Capital & Production

2.1. Healthy Time in the Grossman Model

Much empirical health economics research depends on measuring individuals' health status (Mullahy, 2016). Burns and Mullahy (2016) call time-denominated measures of health as characterizing and measuring "healthy time." The Grossman (1972a) human capital model of the demand for health was the first to formalize the importance of healthy time for individuals. It builds on human capital theory and uses the household production function model by Becker (1965). Grossman's work spawned several theoretical and empirical extensions of his framework (Grossman, 1982). Notably, Rosenzweig and

Schultz (1983) provided a framework to accommodate exogenous health heterogeneity when estimating the effect of determinants of health on health production. For our purposes, Grossman's human capital model is sufficient to motivate the importance of healthy time as an outcome in empirical research.

Grossman's intertemporal utility function for consumer i in a given period t, say a year, is given by

$$U_{it} = U(\phi_{it} H_{it}, Z_{it}),$$

where H_{it} is the health capital stock in year t, Z_{it} denotes the consumption of another commodity (such as leisure) in this year, and ϕ_{it} is the flow of health services in year t per unit of health stock H_{it}. Grossman (2000) assumes $\phi_{it} H_{it}$ to equal healthy time in the year by stating that the health stock provides no services besides healthy time. Consumers produce health H_{it} and other commodities through production functions that take market goods (medical care utilization for health) and allocated time as inputs. This model rules out the joint production of health and other commodities by assuming that inputs, including time, for the production of health H_{it} do not affect the production of other commodities and vice versa. See Grossman (1972b) for discussion about incorporating joint production into the production functions.

Let $\Omega = 365$ be the number of days available in year t. The time budget Ω can be broken into its mutually exclusive component shares $TL_{it} + TH_{it} + T_{it} = \Omega$, where TH is time allocated towards investment in health H (such as time spent exercising), TL is time lost to illness or injury, and T is time devoted to work or the production of other commodities such as leisure activities.[6] Healthy time is the total number of days not ill or injured in the year, i.e., $h_{it} = \Omega - TL_{it}$. The quantity $h_{it} = \phi_{it} H_{it}$ also represents healthy time in year t, when the health stock is assumed to yield only healthy time as a service. In this context, ϕ_{it} represents how productively consumer i can generate healthy time from a unit stock of health in year t. The representation $h_{it} = \phi_{it} H_{it}$ helps clarify the conceptualization of healthy time as a flow in a year that depends on the health stock that year.

Healthy time h_{it} is a consumption commodity that directly enters a person's utility function. It is intuitive that people value healthy time and that time spent ill is a source of disutility (Ganguli, 2024). Health capital H_{it} does not directly affect utility. Time allocated to investment in next-period health $H_{i(t+1)}$ subtracts from time available for work and other activities during this period.

2.2. Upper Bounds on the Distribution of Individual Healthy Time

A hospital-utilization-based measure of healthy time has to contend with the issue that time spent outside the hospital may not necessarily be time spent healthy, but time spent ill. While discussing utilization-based measures of health outcomes, Mullahy (2016) notes that the premise that "a given day within the accounting period has a positive value if the individual is alive and not in contact with the healthcare system on that day … merits scrutiny in some contexts." A day when an individual is not in contact with the healthcare

[6] We depart from Grossman's notation, in which time devoted to work is separately represented as TW.

system has a positive value, according to Grossman's model, if the patient is not ill and can thus allocate that time towards work, improving their health, or other activities that provide utility. Patient-centered outcomes of health status such as *Contact Days* (The ESCAPE Investigators and ESCAPE Study Coordinators, 2005) and *Days Alive Out of Hospital* (Medicare Payment Advisory Committee (MedPAC), 2015; Meza et al., 2024) are operationalized as time spent not in contact with the healthcare system. Nevertheless, healthcare systems in many countries, such as Ireland, have long waiting times to obtain appointments for health services (Whyte et al., 2020). A patient waiting to receive care is likely to be unhealthy for at least a part of their waiting period.

Since time spent outside the hospital is not necessarily healthy, we propose the following rate as an upper bound on healthy time in a year t. In a cohort of N patients for which a researcher has inpatient admission data, let d_{it} denote the follow-up time (time that a patient is observed by the researcher, such as the time that they are enrolled in health insurance) in days for patient i in year t. Let l_{it} denote the total number of days during the follow-up period that patient i is hospitalized. Using the $\Omega - TL_{it}$ definition of healthy time, define the following rate for patient i in year t with 365 days,

$$h_{U_{it}} = \frac{(365 - l_{it})}{365} = 1 - \frac{l_{it}}{365}.$$

This is an upper bound on the proportion of the year that patient i is healthy. If $365 > d_{it}$ and the researcher knows that the patient i has been hospitalized for all the unobserved $365 - d_{it}$ days, then the following rate ${h_{U_{it}}}'$ is an upper bound lower than $h_{U_{it}}$:

$${h_{U_{it}}}' = \frac{365 - l_{it} - (365 - d_{it})}{365} = \frac{d_{it} - l_{it}}{365}.$$

If the patient is hospitalized for l'_{it} unobserved days where $0 < l'_{it} < (365 - d_{it})$, then ${h_U}_{it}'$ is no longer an upper bound. In this case, ${h_U}_{it}$ should be used.

3. Clinical Medicine: Hospital Admissions

3.1. A Common Hospitalization Rate

Measuring the progression of a disease is crucial to clinical medicine. Disease progression refers to changes in the severity of the disease, particularly worsening of the disease over time. Understanding disease progression helps clinicians identify patients at higher risk of illness and tailor treatment accordingly. Hence, clinical researchers are concerned with finding quality metrics for measuring disease progression using real-world data (Amorrortu et al., 2023). Moreover, literature on the value of healthcare has discussed the importance of measuring the severity of disease in economic assessments of healthcare resources (Lakdawalla et al., 2018; Shah, 2009).

Disease progression is sometimes conflated with disease burden. Disease burden refers to the impact of disease on a population or resource system (Udompap et al., 2015). Without a description of the estimand, it is hard to interpret what concept researchers aim to measure. Metcalfe et al. (2003), discussing how clinical researchers could measure hospital admissions when studying heart failure (HF), suggest using only those admissions indicative of HF progression. At the same time, they mention that readmissions informative of disease burden should also be measured. While severe disease poses a more significant burden on the patient and healthcare system, researchers may be able to develop metrics more suitable for measuring disease progression if they are clear about the quantity they want to measure. Which type of hospital stays to include in the measure might differ when measuring disease burden than when measuring disease progression. For instance, only hospitalizations where the disease is the primary cause may be relevant for measuring disease progression. However, all hospitalizations, regardless of the reason, contribute to the burden on the resource system for patients with the disease.

Researchers use admissions data to define various outcome measures, such as the mean number of admissions per patient or the mean number of days spent in the hospital per patient. A common way to define a hospitalization rate is to measure it such that the rate has units of admissions per patient-year of follow-up. Chen et al. (2011) studied a cohort of fee-for-service Medicare beneficiaries who were hospitalized for heart failure (HF). To define the admission rate, they “tabulated the total beneficiary-months at risk (subsequently converted to beneficiary-years) for a given year to use as the denominator, with the total number of HF hospitalizations for a given year as the numerator." Similarly, Davy-Mendez et al. (2019) defined hospitalization rates “as the number of hospitalizations divided by the person-time at risk, for the study period and each calendar year, among all patients and demographic subgroups."

Formally, this hospitalization rate is conventionally measured as

$$H' = \frac{365 \sum_i n_i}{\sum_i d_i},$$

where the summations are over all patients i in the cohort, n_i denotes the number of hospitalizations for patient i over the patient’s follow-up, and d_i denotes the number of days of follow-up for patient i. $\sum_i d_i$ is divided by 365 days to convert total patient-days of follow-up time to patient-years of follow-up.

If we let $L = \sum_i d_i$ be the total amount of follow-up time across all patients in the cohort, then we can write

$$H' \quad = \frac{365 \sum_i n_i}{L} = \frac{365}{L} \sum_i \frac{d_i n_i}{d_i} = \sum_i \frac{d_i}{L} \frac{365\, n_i}{d_i}.$$

Since $\sum_i \frac{d_i}{L} = 1$, H' is a weighted mean of individual hospitalization rates $\left(\frac{365 n_i}{d_i}\right)$ that are weighted by the proportion of total follow-up time observed for each patient. It is not apparent why clinical researchers should want to weigh individual hospitalization rates more greatly for those observed for a longer period in some available dataset.

3.2. Reasons Behind H' and a Proposed Alternative

Metcalfe et al. (2003) note that some studies present the conventional hospitalization rate H' “to counter" the fact that patients are observed for different amounts of time. However, this explanation does not address the specific weighting scheme that is used. They argue that high mortality among ill patients results in shorter follow-up periods for those patients. Perhaps researchers choose to weigh individual rates more greatly for patients observed over longer periods to achieve what they call “comparability" among patients with variable follow-up, based on the belief that patients with shorter follow-up periods have higher hospitalization rates. However, it is unclear what “comparability" among patients means and how “adjustment" of individual rates achieves that.

Another possible reason why H' is commonly used as the admission rate in clinical research is that it has been inspired by incidence rates used in epidemiology research. An incidence rate is the ratio of the number of (incident) events, such as new hospitalizations, to patient-time, such as the total duration of inpatient stays or the total time of enrollment in insurance. In the early 1860s, British statisticians Nightingale and Farr proposed that administrators use hospital-level mortality incidence rates (with total days spent in the hospital as the denominator) to compare sanitary conditions among different hospitals (Cummings, 2019). Nightingale proposed that hospital medical authorities provide information that would allow administrators to calculate the “number of cases" in a year and the “average duration" of an inpatient stay in days (Nightingale, 1862). At the time, it may not have been feasible for administrators to use information on each patient’s hospital stay to manually calculate an individual-level incidence rate first, and therefore, rates were calculated directly using hospital-wide data.

Rather than use H' to measure disease progression, we suggest that clinical researchers use the revised rate H that computes a simple average of individual hospitalization rates across the observed cohort of N patients:

$$H = \frac{1}{N}\sum_{i}^{N} \frac{365\ n_i}{d_i}.$$

This rate gives equal weight to each patient in the period that the patient is observed, rather than giving more weight to patients who are observed for longer periods.

4. Clinical Medicine: Hospital Readmissions

4.1. Hospital Readmissions Reduction Program (HRRP): Background and Incentives

Researchers and administrative agencies concerned with the quality of care in inpatient healthcare facilities need to be able to measure hospital performance to assess and compare hospitals. Comparison between hospitals is complicated by differences in the patient case mix (age and illness severity characteristics) and hospital service mix (procedures and services offered by hospitals) (Horwitz et al., 2012).

The problem of comparing patient outcomes across different hospitals has been discussed by statisticians and epidemiologists since 1860 (Vandenbroucke & Vandenbroucke-Grauls, 1988). Health economists have used econometric techniques to develop measures of the impact of hospital differences on patient outcomes (Doyle et al., 2015, 2019). To compare performance across hospitals serving Medicare patients and adjust for differences in case mix, the US Centers for Medicare & Medicaid Services (CMS) relies on risk-standardized measures.

The CMS uses various hospital-performance measures for its reporting and reform initiatives. It publicly reports its Hospital-Wide All-Cause Readmission (HWR) measure for eligible acute care hospitals as part of its Hospital Inpatient Quality Reporting (IQR) Program. The HWR measure is a hospital-level 30-day risk-standardized readmission rate. The IQR program was mandated by congressional legislation, which required hospitals to submit “data that relate to the quality of care furnished by the hospital" (U.S. Congress, 2003).

The CMS is mandated to “reduce the payments" for “excess readmissions" (U.S. Congress, 2010) as part of its Hospital Readmissions Reduction Program (HRRP). CMS calculates payment reduction using a formula that penalizes hospitals for what are termed excess readmissions. Specifically, the payment reduction formula relies on risk-standardized readmission measures for six conditions or procedures. These readmission measures are calculated for each of the six conditions/procedures, meaning that the index admission must have a discharge diagnosis corresponding to one of these six conditions. However, the readmission itself can be for any unplanned cause. These readmission measures are the only performance measures that are used to penalize hospitals under the HRRP.

Other CMS value-based programs reward or penalize hospitals based on different performance measures. For instance, the Hospital Value-Based Purchasing (VBP) Program uses outcomes such as mortality and spending to reward hospitals with incentive payments for the quality of care they provide.

The readmission measures used in both the IQR program and HRRP are based on 30-day unplanned all-cause readmissions. These measures are created using the same risk-standardization methodology. Therefore, we will use the HWR measure as our running example to refer to the HRRP readmission measures used by the CMS.[7]

[7] See (Centers for Medicare & Medicaid Services 2024) to access methodology reports for the CMS readmission measures.

The 2012 original methodology report for the HWR measure argues that 30 days is a clinically reasonable timeframe for defining a hospital quality measure (Horwitz et al., 2012). The reasoning for this argument is based, in part, on randomized controlled trials that broadly study the effect of discharge planning on readmissions.[8] However, discharge planning seems to be related more to the nature of transitional care and a patient's compliance to care instructions in an outpatient setting post-discharge than to the hospital's quality of care during the patient's inpatient stay. The available studies do not show why the 30-day timeframe makes readmission rates a suitable measure of hospital performance during the inpatient stay. Indeed, research indicates that early readmissions are generally more preventable, while readmissions occurring after seven days within the 30-day post-discharge period tend to be less related to factors during the index hospitalization (Graham et al., 2018).

In so far as the HRRP aims to link payment to quality of care, it is unclear why the payment reduction formula should factor in only readmissions and not mortality rates, which the CMS uses in its Hospital VBP program. Generally, a hospitalization is included in the calculation of the HWR as an index admission if the patient was alive upon discharge and continuously enrolled for 30 days in fee-for-service (FFS) Medicare Part A after discharge. However, patients who die within 30 days after discharge and thus have less than 30 days post-discharge enrollment in Medicare FFS are also eligible for inclusion

[8] See (Coleman et al., 2004; Courtney et al., 2009; Garåsen et al., 2007; Jack et al., 2009; Koehler et al., 2009; Mistiaen et al., 2007; M. Naylor et al., 1994; M. D. Naylor et al., 1999; Stauffer et al., 2011; Voss et al., 2011; Walraven et al., 2002; Weiss et al., 2010) for RCT studies cited in the original report.

(DeBuhr et al., 2024). This means that while an unplanned readmission before death is captured in the HWR, death within 30 days of discharge from the index admission without a readmission counts as a “zero readmission". The death does, however, count towards a 30-day mortality rate. This death could be linked to the quality of care received in the hospital during the index admission.

A clinician may not prioritize preventing an unplanned readmission if it helps reduce the risk of death. Further, the extent to which a readmission indicates poor quality of care varies based on the condition. Higher 30-day readmission rates could even suggest good quality of care for some of the six conditions/procedures for which risk-standardized readmission measures are calculated in the HRRP. For example, heart failure mortality rates are negatively associated with readmissions (Gorodeski et al., 2010). Thus, a clinician's treatment to reduce 30-day mortality risk among patients hospitalized for a specific disease may not align with the goal of reducing 30-day readmission rates. However, this clinician’s treatment would align with the Hospital VBP program because this program uses 30-day mortality (and not readmission) as a clinical outcome.

Both the Hospital VBP program and HRRP apply to the majority of acute care hospitals that receive payment under the Inpatient Prospective Payment System (IPPS). Future research should study how to jointly assess the effect of a bundle of incentives from CMS pay-for-performance programs that use different clinical outcomes to evaluate providers’ behavior and hospital practices.[9] There is likely heterogeneity in how hospitals respond to

[9] Health economists recognize that hospital quality measures are multidimensional and often controversial, which makes it difficult to measure quality (Doyle et al., 2019; Pope,

these various programs. Incentives to improve certain mortality measures in the Hospital VBP program vary across hospitals, with some hospitals having no incentives to improve on specific measures (Norton et al., 2023). It is unclear how clinicians respond to these bundles of incentives when treating diseases or conditions where mortality is negatively associated with readmission.

Some providers have expressed concerns about the proliferation and development of CMS quality measures (Jacobs et al., 2023; Talutis et al., 2019). Clinical researchers have argued that linking quality measures to reimbursement could discourage providers from serving vulnerable populations (Maddox, 2018). This suggests that providers are influenced by various clinical and financial incentives. Indeed, following the implementation of the inpatient prospective payment system, hospitals sought to reduce the average length of stay for Medicare beneficiaries, as the system reimbursed hospitals a fixed amount per hospitalization rather than based on the duration of the stay (Barnett et al., 2017).

4.2. Calculation of the Hospital-Wide All-Cause Readmission (HWR) Rate

CMS calculates the HWR by estimating a hierarchical logistic regression model. This is done as follows. Let Y_{ij} represent a binary outcome for whether a patient i is readmitted for an unplanned readmission at hospital j within 30 days.[10] Let Z_{ij} represent a vector of

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2009). While measures of hospital quality are widely debated, we could not find any paper that discusses how to account for interactions among various quality-improving policies and their associated incentives when measuring hospital quality.

[10] For a list of planned procedure codes used to exclude planned readmissions, see (Centers for Medicare & Medicaid Services 2023).

patient-specific covariates or risk factors. Let α_j be the hospital-specific intercept, assumed to follow a cross-hospital normal distribution with mean μ and variance τ^2. Then, Y_{ij} is related to the covariates as follows:

$$\ln\left(\frac{Pr\left(Y_{ij}=1|Z_{ij},\omega_j\right)}{1-Pr\left(Y_{ij}=1|Z_{ij},\omega_j\right)}\right)=\alpha_j+\beta Z_{ij},$$

where $\alpha_j \quad = \mu + \omega_j$ and $\omega_j \sim N(0,\tau^2)$.

After using this hierarchical logistic regression model to obtain parameter estimates, the ratio of predicted readmissions to the number of expected readmissions is calculated as

$$\widehat{s_j} = \frac{\sum_{i=1}^{n_j} f\left(\widehat{\alpha_j}+\hat{\beta}Z_{ij}\right)}{\sum_{i=1}^{n_j} f\left(\hat{\mu}+\hat{\beta}Z_{ij}\right)},$$

where $f(x) = \frac{1}{1+e^{-x}}$ is the standard logistic function, and n_j denotes the number of index admissions at hospital j. This is the “standardized readmission ratio" for hospital j. It is computed to compare the “observed” performance of a particular hospital to the “expected” performance of an average hospital (DeBuhr et al., 2024). It is multiplied by the national crude readmission rate to yield a risk-standardized readmission rate, which is the HWR measure.

Under this model, the hospital-specific intercepts α_j capture differences in readmissions across hospitals due to unobservable hospital characteristics. The hospital-specific intercept is estimated while conditioning on patient covariates and reflects differences in hospital quality that are independent of the observable case mix. Therefore, we believe that CMS should rank hospitals based on intercepts α_j rather than the standardized readmission ratio described above.

Hospital rankings based on intercepts may differ from those based on standardized readmission ratios. In Appendix A, we formally show that for two hospitals indexed as 1 and 2, each with one patient, and with $\mu > \widehat{\alpha_1} > \widehat{\alpha_2}$, making the patient worse off in Hospital 2 can produce a hospital ranking that is inconsistent with the ranking based on intercepts $\hat{\alpha}$ (Proposition A1). Specifically, Hospital 1 can end up ranking better in performance (lower $\hat{s}$) than Hospital 2 when using $\hat{s}$, despite ranking worse when using $\hat{\alpha}$ (higher $\hat{\alpha}$). In Table 1, we present a numerical example based on this result. Hospital 2 experiences an increase in its standardized readmission ratio when $\hat{\beta}Z_{12}$ is increased, meaning the patient's condition worsens. This increased ratio exceeds that of Hospital 1 when $\hat{\beta}Z_{11}$ equals one, ranking Hospital 2 as a worse performer when its patient is sicker. Since CMS determines payment reductions by comparing a hospital's standardized readmission ratio to the median ratio of peer group hospitals, this example illustrates that a worse patient case-mix at Hospital 2 could lead to a greater payment reduction due to an increased standardized readmission ratio. Thus, hospitals that perform better than average in preventing readmissions may be penalized for a poor case mix when the standardized readmission ratio is used to assess performance.

We prove an analogous result that a hospital performing worse than average in preventing readmissions can, conversely, benefit from a poor case-mix (Appendix A, Proposition A2; Table 2). In Table 2, we show that when two hospitals serving a patient have $\widehat{\alpha_1} > \widehat{\alpha_2} > \mu$, Hospital 1—despite having lower quality (higher $\hat{\alpha}$)—ranks better than

Hospital 2 (lower $\hat{s}$) when the patient case-mix at Hospital 1 worsens (i.e., an increase in $\hat{\beta}Z_{11}$) and performance is assessed using the standardized readmission ratio.

Thus, by using hospital-specific intercepts to rank hospitals, CMS would evaluate them based on their unobserved efficiency in preventing readmissions—the quantity of interest.

4.3. HRRP Effectiveness and Incentives

The HRRP aims to reduce all-cause readmissions broadly—the payment reduction formula used to penalize hospitals aggregates condition-specific standardized readmission ratios for six conditions/procedures. The program does not distinguish between condition-specific challenges to preventing readmissions.

Some policymakers recommend that CMS move away from condition-specific measures and use the HWR measure instead because it is hospital-wide (Zuckerman et al., 2017). However, this could further incentivize hospitals to reduce readmissions more than mortality (Abdul-Aziz et al., 2017; Jha, 2018), depending on the incentives to increase survival from the Hospital VBP program. By penalizing readmissions that help clinicians lower mortality rates among patients with a specific chronic disease, the HRRP may be misaligning hospital and clinician incentives.

Understanding the effectiveness of payment reductions in preventing readmissions is further complicated by the unclear impact of the public reporting of the CMS readmission rates on the quality of healthcare provided by hospitals. According to theoretical and empirical health economics research, the disclosure of health care quality information is likely to affect the behavior of physicians and hospitals (Dranove & Jin, 2010). This literature is divided on whether disclosure would result in better or poorer quality of health care. Physicians and hospitals could try to “game" the reported quality and negatively affect patient outcomes by avoiding sick patients (Dranove et al., 2003). But public readmission rates could also help regulate quality by incentivizing hospitals to produce quality at the national average readmission rate (Vatter, 2024).

Himmelstein and Woolhandler (2015) argue that, in response to the HRRP payment reductions for excessive readmissions, hospitals appear to be treating recently discharged patients in emergency departments without readmitting them. They also seem to be treating these patients in inpatient units while classifying them as being in observation status, preventing them from appearing in inpatient statistics. If true, this practice would leave patients worse off financially and could also affect their health outcomes through decreased quality of care.

The Medicare Payment Advisory Commission (MedPAC), on the other hand, is dismissive of Himmelstein and Woolhandler (2015). Their report (MedPAC, 2018) notes that readmission rates declined between 2010 and 2016, following the implementation of the HRRP, without increasing risk-adjusted mortality. Other clinical research has also argued that these readmission trends are likely due to program incentives (Zuckerman et al., 2016).

Some economists have contributed to the debate, arguing that the HRRP’s impact on reducing readmissions is either negligible or smaller than reported in certain clinical research articles (Ody et al., 2019). A significant share of the reduction in readmissions after the implementation of the HRRP may be due to other factors, such as hospitals increasing the coded severity of patients' illness (Ibrahim et al., 2018). This is because the recorded diagnoses used for risk standardization reflect not only the patient's underlying health status but also the provider's tendency to upcode when making a diagnosis (Finkelstein et al., 2017). On the other hand, Gupta (2021) finds empirical evidence that the program succeeded in motivating better quality of care by hospitals, despite readmissions reductions due to manipulation.

In sum, the effectiveness of the HRRP in reducing readmissions remains uncertain, indicating the need for further research.

5. Conclusion

The same hospitalization rate is sometimes used to measure different quantities in different fields. Readmission rates measure disease progression and hospital performance in clinical research. It is possible for rates to be defined differently but still be referred to by the same name. Readmission rates can refer to the ratio of readmissions to index hospitalizations, but they can also refer to the ratio of predicted readmissions to expected readmissions when assessing hospital performance.

The heuristic use of existing rate definitions in health research underscores our message: researchers should elaborate on the definitions and motivations behind their estimators to help clarify them for readers. Relatedly, the appraisal of estimators should give central importance to the quantity that the researchers aim to measure. Researchers may realize that there are better measures or more than one estimator that they could use to measure the quantity of interest.

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Tables

Table 1: Standardized Readmission Ratios for High-Quality Hospitals Across Patient Risk Factors

Hospital $\hat{\alpha}$	$\hat{s}$ when $\hat{\beta}Z = 1$	$\hat{s}$ when $\hat{\beta}Z = 2.5$
$\hat{\alpha}_1 = 0.9$	**0.988**	0.997
$\hat{\alpha}_2 = 0.7$	0.960	**0.990**

Notes: This table compares the standardized readmission ratio of two hospitals serving a patient when the patient has different risk factors. Both hospitals have $\hat{\alpha}$ values below μ, the average quality, which equals 1. Hospital 1 has a higher $\hat{\alpha}$ and is therefore of lower quality than Hospital 2. Bolded ratio values indicate that an increase in $\hat{\beta}Z$ for the patient in Hospital 2 results in a ratio that exceeds that of Hospital 1 for the original $\hat{\beta}Z$.

Table 2: Standardized Readmission Ratios for Low-Quality Hospitals Across Patient Risk Factors

Hospital $\hat{\alpha}$	$\hat{s}$ when $\hat{\beta}Z = 1$	$\hat{s}$ when $\hat{\beta}Z = 3$
$\hat{\alpha}_1 = 1.5$	1.049	**1.007**
$\hat{\alpha}_2 = 1.1$	**1.011**	1.002

Notes: This table compares the standardized readmission ratio of two hospitals serving a patient when the patient has different risk factors. Both hospitals have $\hat{\alpha}$ values above μ, the average quality, which equals 1. Hospital 1 has a higher $\hat{\alpha}$ and is therefore of lower quality than Hospital 2. Bolded ratio values indicate that an increase in $\hat{\beta}Z$ for the patient in Hospital 1 results in a ratio lower than that of Hospital 2 for the original $\hat{\beta}Z$.

Appendix A

Consider two hospitals indexed by 1 and 2. Each hospital serves one patient. Let $\gamma_i = \hat{\beta} Z_{1i} > 0$ denote the estimated coefficient vector multiplied by the covariate vector for patient 1 at hospital i.

Proposition A1. *If $\widehat{\alpha_2} < \widehat{\alpha_1} < \mu$, then, for a given γ_1, there exists $\gamma_2 > \gamma_1$ such that $\hat{s}_2 > \hat{s}_1$.*

Proof. We will show $\exists\ \gamma_2$ such that $\hat{s}_1 - \hat{s}_2 < 0$.

Write:

$$\hat{s}_1 - \hat{s}_2 = \frac{1 + e^{-(\mu+\gamma_1)}}{1 + e^{-(\widehat{\alpha_1}+\gamma_1)}} - \frac{1 + e^{-(\mu+\gamma_2)}}{1 + e^{-(\widehat{\alpha_2}+\gamma_2)}}.$$

Rewriting,

$$\hat{s}_1 - \hat{s}_2 = \frac{\left(1 + e^{-(\mu+\gamma_1)}\right)\left(1 + e^{-(\widehat{\alpha_2}+\gamma_2)}\right) - \left(1 + e^{-(\mu+\gamma_2)}\right)\left(1 + e^{-(\widehat{\alpha_1}+\gamma_1)}\right)}{\left(1 + e^{-(\widehat{\alpha_1}+\gamma_1)}\right)\left(1 + e^{-\widehat{\alpha_2}+\gamma_2}\right)}.$$

The denominator is always positive. Thus, we denote the numerator N and show $N < 0$.

Now,

$$N = \left(e^{-(\widehat{\alpha_2}+\gamma_2)} + e^{-(\mu+\gamma_1)} + e^{-(\mu+\gamma_1+\widehat{\alpha_2}+\gamma_2)}\right) - \left(e^{-(\widehat{\alpha_1}+\gamma_1)} + e^{-(\mu+\gamma_2)} + e^{-(\mu+\gamma_1+\widehat{\alpha_1}+\gamma_2)}\right).$$

Let

$$\begin{aligned} A &= \left(e^{-(\widehat{\alpha_2}+\gamma_2)} - e^{-(\widehat{\alpha_1}+\gamma_1)}\right) \\ B &= \left(e^{-(\mu+\gamma_1)} - e^{-(\mu+\gamma_2)}\right) \\ C &= \left(e^{-(\mu+\gamma_2+\widehat{\alpha_2}+\gamma_2)} - e^{-(\mu+\gamma_2+\widehat{\alpha_1}+\gamma_2)}\right). \end{aligned}$$

We can write $N = A + B + C$. Pick $\gamma_2 > \gamma_1$ such that $A < -(B + C)$. Then, we have that $N < 0$. This is possible because $A < 0$ and $B > 0$ if $\gamma_2 > \gamma_1$. Having $\mu > \widehat{\alpha_1}, \widehat{\alpha_2}$ allows $A + B < 0$. Further, $C > 0$ always since $\widehat{\alpha_1} > \widehat{\alpha_2}$. □

Proposition A2. *If $\widehat{\alpha_1} > \widehat{\alpha_2} > \mu$, then, for a given γ_2, there exists $\gamma_1 > \gamma_2$ such that $\hat{s}_2 > \hat{s}_1$.*

Proof. We will show $\exists\ \gamma_1$ such that $\hat{s}_1 - \hat{s}_2 < 0$.

Write:

$$\hat{s}_1 - \hat{s}_2 = \frac{1 + e^{-(\mu+\gamma_1)}}{1 + e^{-(\widehat{\alpha_1}+\gamma_1)}} - \frac{1 + e^{-(\mu+\gamma_2)}}{1 + e^{-(\widehat{\alpha_2}+\gamma_2)}}.$$

Rewriting,

$$\hat{s}_1 - \hat{s}_2 = \frac{\left(1 + e^{-(\mu+\gamma_1)}\right)\left(1 + e^{-(\widehat{\alpha_2}+\gamma_2)}\right) - \left(1 + e^{-(\mu+\gamma_2)}\right)\left(1 + e^{-(\widehat{\alpha_1}+\gamma_1)}\right)}{\left(1 + e^{-(\widehat{\alpha_1}+\gamma_1)}\right)\left(1 + e^{-\widehat{\alpha_2}+\gamma_2}\right)}.$$

The denominator is always positive. Thus, we denote the numerator N and show $N < 0$.

Now,

$$N = \left(e^{-(\widehat{\alpha_2}+\gamma_2)} + e^{-(\mu+\gamma_1)} + e^{-(\mu+\gamma_1+\widehat{\alpha_2}+\gamma_2)}\right) - \left(e^{-(\widehat{\alpha_1}+\gamma_1)} + e^{-(\mu+\gamma_2)} + e^{-(\mu+\gamma_1+\widehat{\alpha_1}+\gamma_2)}\right).$$

Let

$$\begin{aligned} A &= \left(e^{-(\widehat{\alpha_2}+\gamma_2)} - e^{-(\widehat{\alpha_1}+\gamma_1)}\right) \\ B &= \left(e^{-(\mu+\gamma_1)} - e^{-(\mu+\gamma_2)}\right) \\ C &= \left(e^{-(\mu+\gamma_2+\widehat{\alpha_2}+\gamma_2)} - e^{-(\mu+\gamma_2+\widehat{\alpha_1}+\gamma_2)}\right). \end{aligned}$$

We can write $N = A + B + C$. Pick $\gamma_1 > \gamma_2$ such that $B < -(A + C)$. Then, $N < 0$. This is possible because $A > 0$ and $B < 0$ if $\gamma_1 > \gamma_2$. Having $\mu < \widehat{\alpha_1}, \widehat{\alpha_2}$ allows $B + A < 0$. Further, $C > 0$ always since $\widehat{\alpha_1} > \widehat{\alpha_2}$. □